Iterative processes for nonlinear problems: from Newton to nowadays

Alicia Cordero, Juan R. Torregrosa

Solving nonlinear problems modeled by the expression $F(x) = 0$, where F is a nonlinear function between two Banach spaces, is both a classical and topical problem that numerous researchers have worked on throughout history and in recent years is being very prolific, in part thanks to the computational tools that are being developed.

In general, there are no analytical methods to solve these problems, so we must use iterative schemes that approximate the solution by starting from an initial estimate. Among these procedures, possibly the best known and most widely used is Newton's method, whose iterative expression is

$$x_{k+1} = x_k - [F'(x_k)]^{-1} F(x_k),\ k = 0, 1, \ldots,$$

where $F'(x_k)$ is the "derivative" of function F evaluated in the iterate x_k. This scheme has quadratic convergence under some conditions.

Throughout history, and especially in recent years, numerous new iterative methods have been designed trying to improve the convergence speed, without increasing too much the computational cost and smoothing the convergence conditions. In this paper we will present an overview of this development from different points of view.

One of these approaches is the use of complex dynamics or multidimensional real dynamics tools to analyze the stability of the different methods, selecting those with good properties and rejecting those with chaotic behavior.